Asynchronous Saddle Point Method: Interference Management Through Pricing

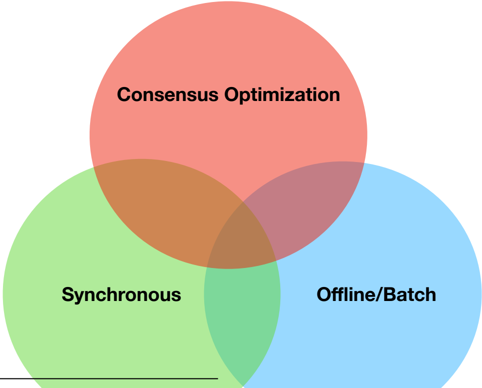
Amrit Singh Bedi [†], Alec Koppel[§], Ketan Rajawat [†]

[†] Dept. of EE, Indian Institute of Technology Kanpur,

[§] CISD, U.S. Army Research Laboratory, alec.e.koppel.civ@mail.mil

Feedback-based Online Optimization for Networked Systems
IEEE Conference on Decision and Control
Miami, FL, Dec. 18, 2018

The State of Distributed Learning



Bertsekas, Dimitri P., and John N. Tsitsiklis. Parallel and distributed computation: numerical methods. Vol. 23. Englewood Cliffs, NJ: Prentice hall, 1989.

A. Nedić and A. Ozdaglar, Distributed subgradient methods for multiagent optimization, IEEE Transactions on Automatic Control

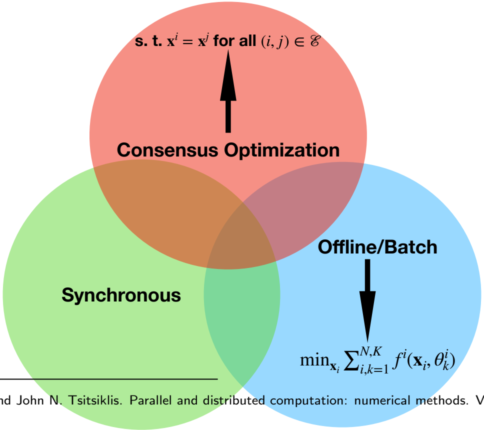
Jakovetić, Dušan and Xavier, Joao and Moura, José MF. "Fast distributed gradient methods." IEEE Transactions on Automatic Control 59.5 (2014): 1131-1146.

Shi, Wei, et al. "Extra: An exact first-order algorithm for decentralized consensus optimization." SIAM Journal on Optimization 25.2 (2015): 944-966.

Mokhtari, Aryan, Qing Ling, and Alejandro Ribeiro. "Network Newton distributed optimization methods." IEEE Transactions on Signal Processing 65.1 (2017): 146-161.

Uribe, Csar A., et al. "A dual approach for optimal algorithms in distributed optimization over networks." arXiv preprint arXiv:1809.00710 (2018).

The State of Distributed Learning



Bertsekas, Dimitri P., and John N. Tsitsiklis. Parallel and distributed computation: numerical methods. Vol. 23. Englewood Cliffs, NJ: Prentice hall, 1989.

A. Nedić and A. Ozdaglar, Distributed subgradient methods for multiagent optimization, IEEE Transactions on Automatic Control

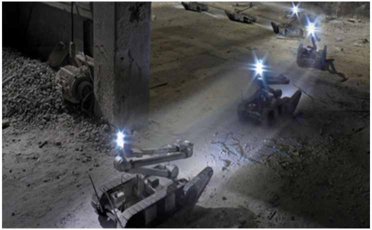
Jakovetić, Dušan and Xavier, Joao and Moura, José MF. "Fast distributed gradient methods." IEEE Transactions on Automatic Control 59.5 (2014): 1131-1146.

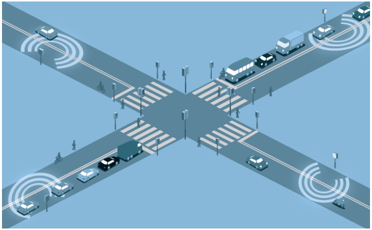
Shi, Wei, et al. "Extra: An exact first-order algorithm for decentralized consensus optimization." SIAM Journal on Optimization 25.2 (2015): 944-966.

Mokhtari, Aryan, Qing Ling, and Alejandro Ribeiro. "Network Newton distributed optimization methods." IEEE Transactions on Signal Processing 65.1 (2017): 146-161.

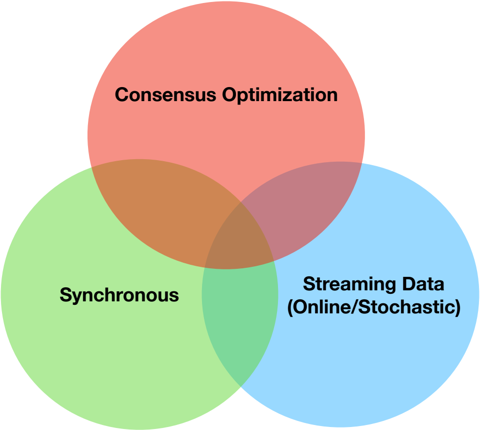
Uribe, Csar A., et al. "A dual approach for optimal algorithms in distributed optimization over networks." arXiv preprint arXiv:1809.00710 (2018).

Modern Networked Autonomy





The State of Distributed Learning



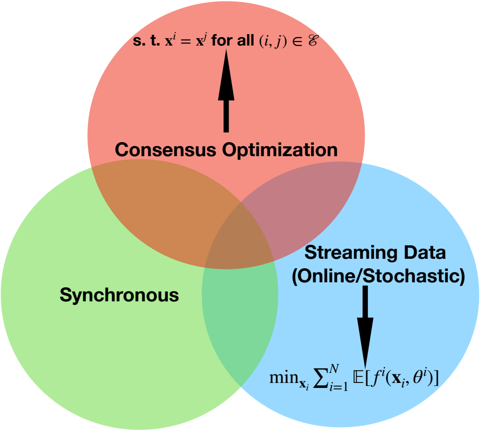
De, Soham, and Tom Goldstein. "Efficient distributed SGD with variance reduction." 2016 IEEE 16th International Conference on Data Mining (ICDM). IEEE, 2016.

"Communication-efficient algorithms for decentralized and stochastic optimization, " Lan, G., Lee, S. & Zhou, Y. Mathematical Programming. (2018).

Mokhtari, Aryan, and Alejandro Ribeiro. "DSA: Decentralized double stochastic averaging gradient algorithm." JMLR 2015

Nokleby, Matthew, and Waheed U. Bajwa. "Stochastic optimization from distributed, streaming data in rate-limited networks." IEEE Transactions on Signal and Information Processing over Networks (2018).

The State of Distributed Learning



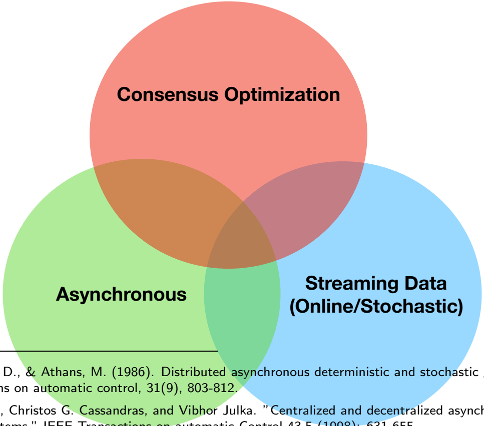
De, Soham, and Tom Goldstein. "Efficient distributed SGD with variance reduction." 2016 IEEE 16th International Conference on Data Mining (ICDM). IEEE, 2016.

"Communication-efficient algorithms for decentralized and stochastic optimization, " Lan, G., Lee, S. & Zhou, Y. Mathematical Programming. (2018).

Mokhtari, Aryan, and Alejandro Ribeiro. "DSA: Decentralized double stochastic averaging gradient algorithm." JMLR 2015

Nokleby, Matthew, and Waheed U. Bajwa. "Stochastic optimization from distributed, streaming data in rate-limited networks." IEEE Transactions on Signal and Information Processing over Networks (2018).

The State of Distributed Learning



Tsitsiklis, J., Bertsekas, D., & Athans, M. (1986). Distributed asynchronous deterministic and stochastic gradient optimization algorithms. IEEE transactions on automatic control, 31(9), 803-812.

Vazquez-Abad, Felisa J., Christos G. Cassandras, and Vibhor Julka. "Centralized and decentralized asynchronous optimization of stochastic discrete-event systems." IEEE Transactions on automatic Control 43.5 (1998): 631-655.

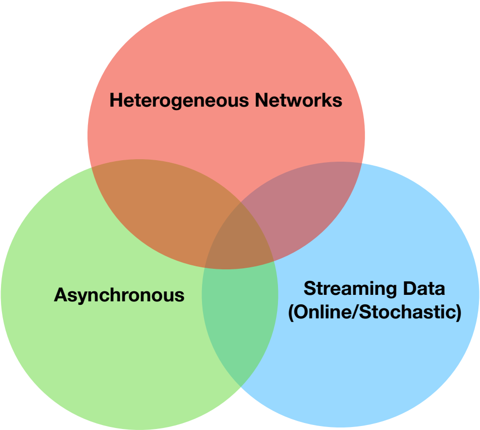
Ram, S. Sundhar, A. Nedić, and Venugopal V. Veeravalli. "Asynchronous gossip algorithms for stochastic optimization." Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on. IEEE, 2009.

Dimakis, A. G., Kar, S., Moura, J. M., Rabbat, M. G., & Scaglione, A. (2010). Gossip algorithms for distributed signal processing. Proceedings of the IEEE, 98(11), 1847-1864.

Lian, X., Zhang, W., Zhang, C., & Liu, J. (2017). Asynchronous Decentralized Parallel Stochastic Gradient Descent. arXiv preprint arXiv:1710.06952.

Sirb, B., Ye, X. (2018). Decentralized consensus algorithm with delayed and stochastic gradients. SIAM Journal on Optimization.

The State of Distributed Learning

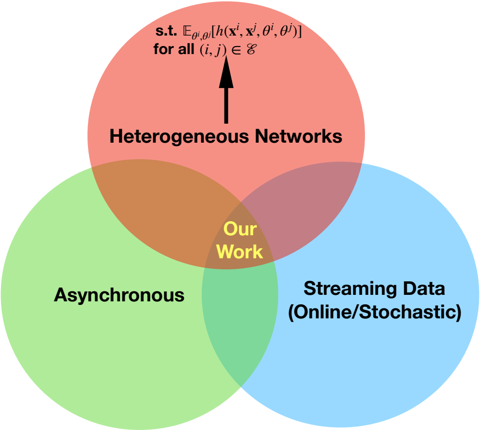


Chen, J., Richard, C. and Sayed, A. H. (2014). Multitask diffusion adaptation over networks. IEEE Transactions on Signal Processing

Nassif, R., Richard, C., Ferrari, A. and Sayed, A. H. Distributed learning over multitask networks with linearly related tasks.

A. Koppel, B. Sadler, and A. Ribeiro, "Proximity without Consensus in Online Multi-Agent Optimization," in IEEE Trans. Signal Process., Mar. 2017

The State of Distributed Learning



Chen, J., Richard, C. and Sayed, A. H. (2014). Multitask diffusion adaptation over networks. IEEE Transactions on Signal Processing

Nassif, R., Richard, C., Ferrari, A. and Sayed, A. H. Distributed learning over multitask networks with linearly related tasks.

A. Koppel, B. Sadler, and A. Ribeiro, "Proximity without Consensus in Online Multi-Agent Optimization," in IEEE Trans. Signal Process., Mar. 2017

Problem Formulation

- Function at each node i is parameterized by random variable $\boldsymbol{\theta}^i$
- Convex objective at each node is denoted by $\mathbb{E}_{\boldsymbol{\theta}_i}[f^i(\mathbf{x}_i, \boldsymbol{\theta}_i)]$
- Optimization problem without consensus is formulated as

$$\mathbf{x}^* \in \underset{\mathbf{x} \in \mathcal{X}^N}{\operatorname{argmin}} \sum_{i=1}^{N} \mathbb{E}_{\boldsymbol{\theta}_i}[f^i(\mathbf{x}^i, \boldsymbol{\theta}^i)]$$

$$\text{s.t.} \quad \mathbb{E}_{\boldsymbol{\theta}^i, \boldsymbol{\theta}^j}\left[h^{ij}(\mathbf{x}^i, \mathbf{x}^j, \boldsymbol{\theta}^i, \boldsymbol{\theta}^j)\right] \leq \gamma_{ij}, \text{ for all } j \in n_i.$$

- Important in the context of communication systems
  - $\Rightarrow$ Interference management in wireless systems
  - $\Rightarrow$ Coordinated beam-forming in cellular network

Constraints Arising in Practice

The constraints in the problem includes

- Consensus constraints

$$\left\|\mathbf{x}^i - \mathbf{x}^j\right\| \leq \gamma_{ij}$$

- Quality of service

$$\mathbf{SINR}(\mathbf{x}^i, \mathbf{x}^j) \geq \gamma_{ij}$$

$\Rightarrow$ where SINR is the signal-to-interference-plus-noise function

- Relative entropy constraint

$$D(\mathbf{x}^i \,||\, \mathbf{x}^j) \leq \gamma_{ij}$$

- Budget constraints

$$\gamma_{ij}^{\min} \leq x^i + x^j \leq \gamma_{ij}^{\max}$$

Contribution

- We propose a stochastic primal-dual method to solve this problem
  - $\Rightarrow$ amenable to asynchronous implementation
  - $\Rightarrow$ consider asynchronous modification
- Mean convergence from synchronous case carries over to this setting
  - $\Rightarrow$ $\mathcal{O}(\sqrt{T})$ primal sub-optimality and $\mathcal{O}(T^{3/4})$ constraint violation
  - $\Rightarrow$ when asynchrony is uniformly bounded
- Demonstrate effective implementation in wireless comms.
  - $\Rightarrow$ experimental benefits of asynchronous online processing

Lagrangian Relaxation

- Consider Lagrangian relaxation of the problem as

$$\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}) = \sum_{i=1}^{N} \left[\mathbb{E}\left[f^i(\mathbf{x}^i, \boldsymbol{\theta}^i) \sum_{j \in n_i} \lambda^{ij} \left(h^{ij} \left(\mathbf{x}^i, \mathbf{x}^j, \boldsymbol{\theta}^i, \boldsymbol{\theta}^j \right) - \gamma_{ij} \right) - \frac{\delta\epsilon}{2} (\lambda^{ij})^2 \right] \right]$$

$\Rightarrow \lambda^{ij} \geq$ $\Rightarrow$ Lagrange multiplier associated to non-linear constraint

- Stochastic approximation of the Lagrangian is

$$\hat{\mathcal{L}}_t(\mathbf{x},\boldsymbol{\lambda}) = \sum_{i=1}^{N}\left[f^i(\mathbf{x}^i,\boldsymbol{\theta}^i_t)\sum_{j\in n_i}\lambda^{ij}\Big(h^{ij}\left(\mathbf{x}^i,\mathbf{x}^j,\boldsymbol{\theta}^i_t,\boldsymbol{\theta}^j_t\right)-\gamma_{ij}\Big)-\frac{\delta\epsilon}{2}(\lambda^{ij})^2\right].$$

$\Rightarrow$ Note the t index
$\Rightarrow$ Objective is stationary here (ensemble average = actual mean)

Stochastic Primal-Dual Method

- Alternate primal descent/dual ascent steps on stochastic Lagrangian

$$\mathbf{x}_{t+1} = \mathcal{P}_{\mathcal{X}}\left[\mathbf{x}_t - \epsilon \nabla_{\mathbf{x}} \hat{\mathcal{L}}_t(\mathbf{x}_t, \lambda_t)\right],$$

$$\lambda_{t+1} = \left[\lambda_t + \epsilon \nabla_{\lambda} \hat{\mathcal{L}}_t(\mathbf{x}_t, \lambda_t)\right]_+,$$

$\Rightarrow$ These updates exhibits decentralized implementation

- Synchronous primal updates is given by

$$\mathbf{x}^i_{t+1} = \mathcal{P}_{\mathcal{X}}\left[\mathbf{x}^i_t - \epsilon\left(\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i_t, \boldsymbol{\theta}^i_t) + \sum_{j\in n_i}\left(\lambda^{ij}_t + \lambda^{ji}_t\right) \nabla_{\mathbf{x}^i} h^{ij}\left(\mathbf{x}^i_t, \mathbf{x}^j_t, \boldsymbol{\theta}^i_t, \boldsymbol{\theta}^j_t\right)\right)\right].$$

Likewise, the dual update for each edge $(i, j) \in \mathcal{E}$ is

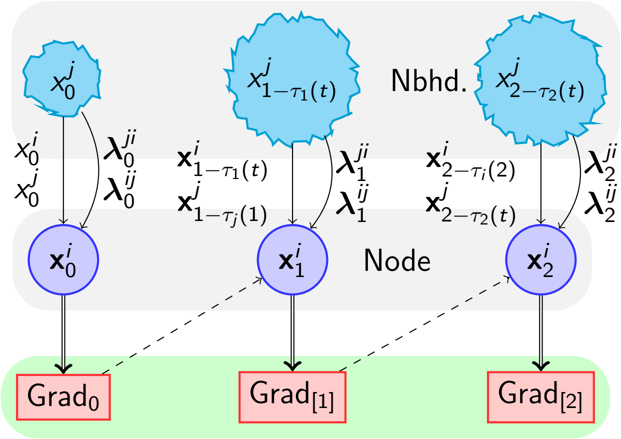
$$\lambda^{ij}_{t+1} = \left[(1 - \epsilon^2\delta)\lambda^{ij}_t + \epsilon\left(h^{ij}\left(\mathbf{x}^i_t, \mathbf{x}^j_t, \boldsymbol{\theta}^i_t, \boldsymbol{\theta}^j_t\right)\right)\right]_+.$$

- The primal update is given by

$$\mathbf{x}_{t+1}^{i} = \mathcal{P}_{\mathcal{X}}\Big[\mathbf{x}_{t}^{i} - \epsilon\Big(\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i_{t-\tau_i(t)}, \boldsymbol{\theta}^i_{t-\tau_i(t)}) + \sum_{j\in n_i}\Big(\lambda_t^{ij} + \lambda_t^{ji}\Big)\nabla_{\mathbf{x}^i} h^{ij}\Big(\mathbf{x}^i_{t-\tau_i(t)}, \mathbf{x}^j_{t-\tau_j(t)}, \boldsymbol{\theta}^i_{t-\tau_i(t)}, \boldsymbol{\theta}^j_{t-\tau_j(t)}\Big)\Big)\Big].$$

- Likewise, the dual update for each edge $(i, j) \in \mathcal{E}$ is

$$\lambda_{t+1}^{ij} = \Big[(1 - \epsilon^2\delta)\lambda_t^{ij} + \epsilon\Big(h^{ij}\Big(\mathbf{x}^i_{t-\tau_i(t)}, \mathbf{x}^j_{t-\tau_j(t)}, \boldsymbol{\theta}^i_{t-\tau_i(t)}, \boldsymbol{\theta}^j_{t-\tau_j(t)}\Big)\Big)\Big]_+.$$


x^j_0
$x^j_{1-\tau_1(t)}$
Nbhd.
$x^j_{2-\tau_2(t)}$
x^i_0
x^j_0
$\boldsymbol{\lambda}^{ji}_0$
$\boldsymbol{\lambda}^{ij}_0$
$\mathbf{x}^i_{1-\tau_1(t)}$
$\mathbf{x}^j_{1-\tau_j(1)}$
$\boldsymbol{\lambda}^{ji}_1$
$\boldsymbol{\lambda}^{ij}_1$
$\mathbf{x}^i_{2-\tau_i(2)}$
$\mathbf{x}^j_{2-\tau_2(t)}$
$\boldsymbol{\lambda}^{ji}_2$
$\boldsymbol{\lambda}^{ij}_2$
$\mathbf{x}^i_0$
$\mathbf{x}^i_1$
Node
$\mathbf{x}^i_2$
Grad_0
$\text{Grad}_{[1]}$
$\text{Grad}_{[2]}$

Technical Settinng

- Assumption 1 Network $\mathcal{G}$ is symmetric and connected
- Assumption 2 Existence of constrained optima (Slater's condition)
- Assumption 3 Stochastic Gradient Variance all i and t satisfy

$$\mathbb{E}\left\|\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i, \boldsymbol{\theta}_t^i)\right\|^2 \leq \sigma_f^2$$

$$\mathbb{E}\left\|\nabla_{\mathbf{x}^i} h^{ij}\left(\mathbf{x}^i, \mathbf{x}^j, \boldsymbol{\theta}^i_{[t]_i}, \boldsymbol{\theta}^j_{[t]_j}\right)\right\|^2 \leq \sigma_h^2$$

- Assumption 4 Constraint has bounded Variance for $(i,j) \in \mathcal{E}$ and t

$$\max_{(\mathbf{x}^i,\mathbf{x}^j)\in\mathcal{X}} \mathbb{E}\left[\left(h^{ij}\left(\mathbf{x}^i, \mathbf{x}^j, \boldsymbol{\theta}^i_t, \boldsymbol{\theta}^j_t\right)^2\right)\right] \leq \sigma_\lambda^2$$

- Assumption 5 Lipschitz continuity

$$\|F(\mathbf{x}) - F(\mathbf{y})\| \leq L_f \|\mathbf{x} - \mathbf{y}\|. \text{ for any } (\mathbf{x}, \mathbf{y}) \in \mathbb{R}^{Np}$$

Assumption 6 (Bounded Delay) for each i, $\tau_i(t) \leq \tau < \infty$

Theorem 1

Under the Assumptions 1-6, with constant step size $\epsilon = 1/\sqrt{T}$, the average time aggregation of the sub-optimality sequence $\mathbb{E}\left[F(\mathbf{x}_t) - F(\mathbf{x}^*)\right]$ is

$$\sum_{t=1}^{T} \mathbb{E}[F(\mathbf{x}_t) - F(\mathbf{x}^*)] \leq \mathcal{O}(\sqrt{T}) \quad (1)$$

Likewise, the delayed time aggregation of the average constrain violation also grows sublinearly in T as

$$\sum_{(i,j)\in\mathcal{E}} \mathbb{E}\left[\left[\sum_{t=1}^{T}\left(h^{ij}(\mathbf{x}^i_{[t]_i}, \mathbf{x}^j_{[t]_j}, \boldsymbol{\theta}^i_{[t]_i}, \boldsymbol{\theta}^j_{[t]_j}) - \gamma_{ij}\right)\right]_+\right] \leq \mathcal{O}(T^{3/4}) \quad (2)$$

- Consider the system model

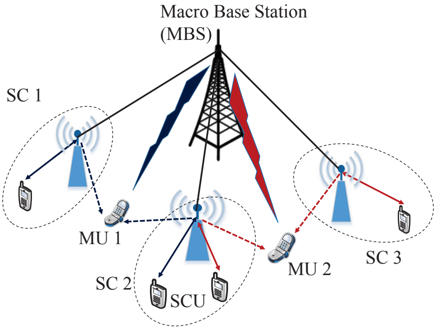


Figure: Heterogeneous cellular network with one MBS, two MUs, and three SCBSs with each serving one, two, and one SCU, respectively.

- BS regulates this cross-tier interference
  $\Rightarrow$ by imposing a penalty x_n^i on the SCBSs $n \in \mathcal{N}_i$
- The total revenue generated by the BS is therefore given by

$$\sum_{i=1}^{M} \sum_{n \in \mathcal{N}_i} x_n^i g_{ni} p_n^i$$

- The BS also adheres to the constraint
  $\Rightarrow$ total penalty imposed on each SCBS is within certain limit, i.e.,

$$C_{\min} \leq \sum_{i : n \in \mathcal{N}_i} x_n^i \leq C_{\max}.$$

  $\Rightarrow$ constraint imposes fairness of base station to small cell operators

Problem Formulation

- The optimization problem for interference management is

$$\max_{\{x_n^i\}} \sum_{i=1}^{M} \sum_{n \in \mathcal{N}_i} \mathbb{E}\left[x_n^i g_{ni} p_n^i(x_n^i, h_n^i)\right]$$

$$\text{s. t.} \quad \sum_{n \in \mathcal{N}_i} \mathbb{E}\left[g_{ni} p_n^i(x_n^i, h_n^i)\right] \leq \gamma_i \;\; 1 \leq i \leq M$$

$$C_{\min} \leq \sum_{i : n \in \mathcal{N}_i} x_n^i \leq C_{\max} \quad 1 \leq n \leq N$$

Proposed Algorithm

Online interference management through pricing

Input initialization $\mathbf{x}_0$ and $\lambda_0 = \mathbf{0}$, step-size ϵ, regularizer δ

for $t = 1, 2, \ldots, T$

loop in parallel for all MU and SCBS user

(1) Send dual vars. λ_t^m to nbhd.

(2) Observe the delayed primal and dual (sub)-gradients

(3) Update the price $x^i_{n,t+1}$ at SCBS n as

$$x^i_{n,t+1} = \mathcal{P}_{\mathcal{X}_n}\Big[x^i_{n,t} + \epsilon\Big(g_{ni,[t]}\left[\frac{W(c\mu_n + \nu_n\lambda^i_t)}{(c\mu_n + \nu_n x^i_{n,[t]})^2} - \frac{1}{h^i_{n,[t]}}\right]\cdot\mathbf{1}(x^i_{n,[t]})\Big)\Big]$$

(4) Update dual variables at each MU i

$$\lambda^i_{t+1} = \left[(1-\delta\epsilon^2)\lambda^i_t - \epsilon\left(\gamma_i - \sum_{n\in\mathcal{N}_i} g_{ni,[t]}\left(\frac{W}{c\mu_n + \nu_n x^i_{n,[t]}} - \frac{1}{h^i_{n,[t]}}\right)_+\right)\right]_+ .$$

end loop

end for

Simulation Results

- **Parameters selection:** For the simulation purposes
  $\Rightarrow$ a cell network w/ $M = 2$ MBSs, $N = 3$ SCBSs
  $\Rightarrow$ {s1, s2} $\Rightarrow$ nbhd of MU $m1$, $\{s2, s3\}$ $\Rightarrow$ nbhd of MU $m2$
  $\Rightarrow$ Random gains g_{ni} and h_n^i $\Rightarrow$ exponential dist. w/ mean $\mu = 3$
  $\Rightarrow$ $C_{\min} = 0.9$ and $C_{\max} = 20$
  $\Rightarrow$ Other parameter values are $W = 1MHz$, $\gamma_i = -3$ dB,
  $\Rightarrow$ $\delta = 10^{-5}$, $c = 0.1$, $\mu_n = \nu_n = 1$, and $\epsilon = 0.01$
  $\Rightarrow$ The maximum delay parameter is $\tau = 10$

Simulation Results

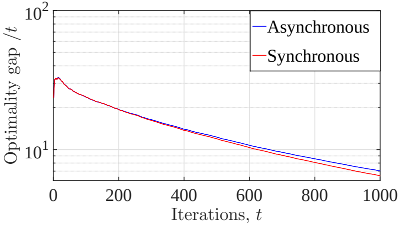


Figure: Average objective sub-optimality vs. iteration t

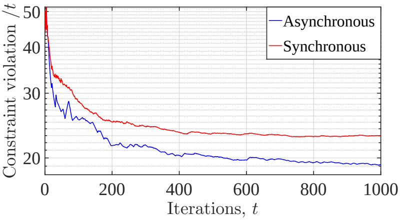


Figure: Average constraint violation vs. iteration t

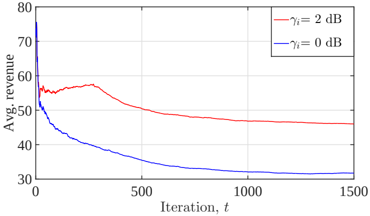


Figure: Average revenue for different interference power margin.

Simulation Results

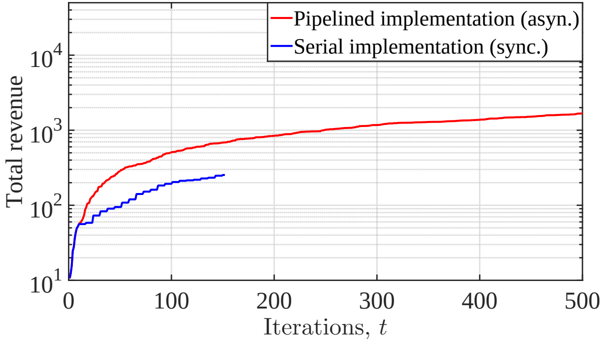


Figure: Total revenue generated for synchronous and asynchronous algorithms.

Conclusion

- Addressed on online learning problems in multi-agent networks
  - $\Rightarrow$ focused on the case where agents' losses are not the same
  - $\Rightarrow$ how to balance local regret with coordination incentives
  - $\Rightarrow$ when nodes do not operate on common synchronized clock
- Proposed a new asynchronous online saddle point algorithm
  - $\Rightarrow$ **sublinear growth** of Delayed regret, Network Discrepancy
  - $\Rightarrow$ **Convergence to Optimal** in stochastic settings
- Online asynchronous vision-based localization with moving cameras
  - $\Rightarrow$ Obtain stable learning in practice, outperform local-only learning
- Online asynchronous interference management through pricing
- Future: beyond vector-valued decisions (nonlinear statistical models)

Thank You